Unknown Input Method Based Observer Synthesis for A Discrete Time Uncertain T-S Fuzzy System

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***Abstract*—In this paper, we propose a novel approach based on unknown input method to synthesize the observer for a discrete uncertain T-S fuzzy system. The uncertainty bounds of the system can be unknown and the premise variables can be measurable or be unmeasurable. Two main theorems are derived based on the non-common quadratic Lyapunov function and feasible parameters are solved by linear matrix inequality (LMI) technique. Finally, a practical example is given to prove the effectiveness of the proposed method.**

***Index Terms*— Discrete time uncertain T-S fuzzy system, observer synthesis, unknown inputs, unmeasurable premise variables, LMIs.**

I. Introduction

IN recent decades, the Takagi-Sugeno (T-S) fuzzy model [1] has been widely applied to deal with nonlinear system control problems. Via various approaches, numerous fuzzy control problems of continuous or discrete time systems in term of stability analysis, stabilization, observer synthesis, controller design and so on have been studied in [2]-[8]. In practice, the state variables of physical systems are not always measurable, or in some cases, the sensors for measuring them are very expensive or difficult to maintain, but these state variables are necessary for controller design or system supervision. Because of these reasons, the observer synthesis for a control system is essential. In past decades, there have been many approaches, such as [4] and [7]-[27], focusing on observer synthesis for T-S fuzzy systems.

Nevertheless, in practice, most systems are impacted by uncertain terms. With the presence of uncertainties, the observer synthesis for T-S fuzzy systems becomes very difficult. In order to eliminate the effects of the uncertainties, several approaches have been developed in past few years. Among of them, the observer-based controller was needed, where the observer and controller were built simultaneously [28]-[34]. However, the uncertainties in these studies must be bounded and satisfy some specific assumptions. Additionally, the sliding mode observer was synthesized to estimate the state variables for a T-S fuzzy system with uncertainties [35], but their uncertainties must fulfill a very strict constraint and the bounded conditions of the uncertainties must be given in advance. In the paper [36], the influence of the uncertainties was handled by adding extra parts to the conventional Luenburger observer, but it needs a strict constraint (19) in [36] for the uncertainties.

It is known that the unknown input method for observer design was introduced in many papers [22]–[27] and [47], however, this method is seldomly applied for uncertain systems. Recently, a new approach for the observer synthesis based on the unknown input method was developed in [38] to estimate the state variables of an uncertain T-S fuzzy system. Unfortunately, it only considered the case of measurable premise variables. Furthermore, the systems considered in [28]-[32] and [35]-[38] are all continuous time systems. After reviewing the above related papers, it is seen that the uncertainties of all previous studies must satisfy the bounded conditions and some strict constraints. Otherwise, it cannot find feasible parameters to synthesize the observer for an uncertain T-S fuzzy system. Because of the above limitations, this study is motivated to propose a new approach to synthesize observer for the discrete time uncertain T-S fuzzy system without bounded conditions and with less conservative conditions. First, let uncertainties be transformed into the unknown input, then the unknown input method is employed to synthesize the observer for T-S fuzzy system with uncertainties.

In addition, numerous recent papers focused on synthesizing observers or observer-based controller for T-S fuzzy systems which have unmeasurable premise variables [26], [35], [39]-[43], and [51]. When the premise variables depend on un-measurable states, it is infeasible to design an observer to estimate states for these systems. Therefore, observer synthesis for the premise variables dependent on measurable state and estimated state variables are also investigated, respectively, in this work. Moreover, non-common quadratic Lyapunov functions [52]-[53] are chosen to design the observer rather than a common Lyapunov so that the derived results will be not so conservative. The main contributions that are worth being emphasized are as follows. 1) The observer synthesis for a discrete time uncertain T-S fuzzy system without bounded constraints of the uncertainties and the impacts of uncertainties are eliminated without using the methods in previous papers [28]-[37], [39] and [40]; and 2) the premise variables of the T-S fuzzy system may be either measurable or need to be estimated.

The rest of this paper is organized as follows. In Section II, we describe the considered fuzzy system model with uncertainties and point out the main problems to be investigated. In Section III, the main theorems and observer synthesis procedures are proposed. In Section IV, a practical example is shown to illustrate the effectiveness of the synthesized observer. Finally, a conclusion is presented in Section V.

Notations: In this paper, $A > 0$ denotes the positive definite matrix A. A^T denotes the transpose of the matrix A; A^{-1} denotes the inverse of A; I denotes an identity matrix. A^+ denotes the Moore-Penrose pseudo-inverse of A with $A^+ = (A^T A)^{-1} A^T$. The symbol $\Re^{n\times m}$ denotes the set of $n\times m$ matrices. The asterisk symbol (∗) denotes the transposed elements of matrices in symmetric positions.

II. System Model and Problem Description

A. System Model

Let us consider a discrete-time nonlinear system as follows

$$x(k+1) = f(x(k), u(k)) \tag{1}$$

where $x(k)$ is the state variables, *f* is the nonlinear function, and $u(k)$ is the input signal. In order to transform the nonlinear system (1) into a T-S fuzzy system, the sector nonlinearity or local approximation method [44] can be applied, and we assume that the output response of the system (1) is with linear form, then this discrete time system (1) can be transformed into a discrete time T-S system with uncertainties as follows,

Rule i:

IF $\theta_1(k)$ is Q_{i1}, …, and $\theta_s(k)$ is Q_{is}, THEN

$$x(k+1) = (A_i + \Delta A_i(k))x(k) + (B_i + \Delta B_i(k))u(k) \tag{2a}$$

$$y(k) = Cx(k),\ i = 1,\ 2, ...,\ r, \tag{2b}$$

where $x(k)\in\Re^n$, $u(k)\in\Re^m$, and $y(k)\in\Re^p$ are the state, input, and output vectors, respectively. θ_i, $i = 1,\ 2,\ ...,\ s,$ is the vector of the premise variable, Q_{ij}, i=1, 2, ..., r and j= 1, 2, ..., s, is the fuzzy set, r is the number of IF-THEN rules. The matrices $A_i\in\Re^{n\times n}$, $B_i\in\Re^{n\times m}$, and $C\in\Re^{p\times n}$ are known constant matrices of the state, input, and output, respectively. Suppose that each pair (A_i, C) is observable. Moreover, $\Delta A_i(k)\in\Re^{n\times n}$ and $\Delta B_i(k)\in\Re^{n\times m}$ are the uncertainties of A_i and B_i, respectively, and they occur from localization errors or from the original nonlinear function *f*. Thus, the overall discrete time T-S fuzzy system with uncertainties from (2) is expressed as follows,

$$\begin{cases} x(k+1) = \sum_{i=1}^{r} \beta_i(\theta(k))\left[(A_i + \Delta A_i(k))x(k) + (B_i + \Delta B_i(k))u(k)\right], \\ y(k) = Cx(k),\ i = 1,\ 2,\ ...,\ r, \end{cases} \tag{3}$$

where $\beta_i(\theta(k)) = w_i(\theta(k)) \Big/ \sum_{i=1}^{r} w_i(\theta(k))$, $w_i(\theta(k)) = \prod_{j=1}^{s} Q_{ij}(\theta_j(k))$, $Q_{ij}(\theta_j(k))$ is the grade of the membership function of $\theta_j(k)$ in Q_{ij},

$$\beta_i(\theta(k)) \ge 0, \text{ and } \sum_{i=1}^{r} \beta_i(\theta(k)) = 1. \tag{4}$$

B. Problem Description

Consider the discrete time T-S fuzzy system with uncertainties in (3) in which all or some state variables of the system are un-measurable. The objective of this paper is to synthesize an observer for estimating these un-measurable state variables. Based on a survey of the existing studies such as [28]-[37], the Luenburger observer model is the most popular observer form for estimating the state variables of T-S fuzzy systems. However, the Luenburger observer is difficult to deal with uncertainties within the system. Let us show the details as follows. The Luenburger observer is as the form

$$\begin{cases} \hat{x}(k+1) = \sum_{i=1}^{r} \beta_i(\theta(k))\left[A_i\hat{x}(k) + B_i u(k) + G_i(y(k) - \hat{y}(k))\right] \\ \hat{y}(k) = C\hat{x}(k),\ i = 1,\ 2,\ ...,\ r, \end{cases} \tag{5}$$

where $\hat{x}(k)$ is the estimated state, $\hat{y}(k)$ is the estimated output value, and G_i is the gain of the observer. The estimation error is denoted by $e(k) = x(k) - \hat{x}(k)$, then the estimation error dynamic is expressed as follows,

$$e(k+1) = x(k+1) - \hat{x}(k+1) \tag{6}$$

Substituting (3) and (5) into (6) yields

$$\begin{aligned} e(k+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} \beta_i(\theta(k))\beta_j(\theta(k))\big[(A_i - G_i C)e(k) \\ + \Delta A_i(k)x(k) + \Delta B_i(k)u(k)\big] \end{aligned} \tag{7}$$

It is noted that $\Delta A_i(k)x(k)$ and $\Delta B_i(k)u(k)$ in (7) may cause finding the gain G_i to be infeasible so that the estimation error $e(k)$ can approach zero asymptotically.

There have been several methods developed in [28]-[37] to overcome the uncertainty problems. Unfortunately, those papers need the bounded conditions of the uncertainties and/or some other strict constraints. Additionally, in practice, it is possible that the premise variables of the observer cannot be measurable either. Therefore, the above limitations motivate us to investigate the observer synthesis for T–S fuzzy systems with uncertainties under the following two cases, respectively.

A) Design an observer with measurable premise variables. B) Design an observer with estimated premise variables. This paper will propose a new approach based on unknown input method to synthesize an observer for each case, such that the state variables of the discrete time T-S fuzzy system with uncertainties (3) are estimated asymptotically.

III. Observer Synthesis

Before carrying out observer synthesis, the uncertain T-S fuzzy system will be transformed into unknown input T-S fuzzy systems. In order to transform the uncertain terms into unknown inputs, the uncertainties must satisfy the following two assumptions.

Assumption 1: There exists a matrix $D \in \Re^{n\times q}$ with $q \le p$ satisfying the matching conditions $\Delta A_i(k) = D\Delta\tilde{A}_i(k)$ and $\Delta B_i(k) = D\Delta\tilde{B}_i(k)$, where D is a full column rank constant matrix, $\Delta\tilde{A}_i(k) \in \Re^{q\times n}$ and $\Delta\tilde{B}_i(k) \in \Re^{q\times m}$ are uncertain matrices.

Remark 1: The uncertainties in papers [28]-[37] must satisfy the assumption $\Delta A_i(k) = D_{ai}\Delta\tilde{A}_i(k)E_{ai}$, $\Delta B_i(k) = D_{bi}\Delta\tilde{B}_i(k)E_{bi}$ and the bounded condition $\Delta\tilde{A}_i^T(k)\Delta\tilde{A}_i(k) \le \mathrm{I}$, $\Delta\tilde{B}_i^T(k)\Delta\tilde{B}_i(k) \le \mathrm{I}$. Here, Assumption 1 does not need the bound of the uncertainties. In addition, this assumption allows us to transform uncertainties into unknown inputs.

Assumption 2: The matrices C and D are full row and column ranks, respectively, and the rank of (CD) is equal to the rank of D.

Remark 2: Assumption 2 is to guarantee the existence of the general solution of matrix linear equations that will be seen in the proofs of Theorem 1 and Theorem 2.

Under Assumption 1, let us define $\nu_i(k) = \Delta\tilde{A}_i(k)x(k)$, $\gamma_i(k) = \Delta\tilde{B}_i(k)u(k)$, and $\omega_i(k) = \nu_i(k) + \gamma_i(k)$, (3) can be written as follows:

$$\begin{cases} x(k+1) = \sum_{i=1}^{r}\beta_i(\theta(k))[A_i x(k) + B_i u(k) + D\omega_i(k)] & \text{(8a)} \\ y(k) = Cx(k),\ i = 1,\ 2,\ ...,\ r. & \text{(8b)} \end{cases}$$

Therefore, the observer will be synthesized for the system (8) instead of the system (3). In terms of observer design for (8), there are two cases to be taken into account. The first case is with measurable premise variables, and the second is with estimated premise variables.

A. Observer Synthesis with Measurable Premise Variables

Suppose the premise variables of (8) are measurable. Let the observer form for the system (8) be

$$\begin{cases} z(k+1) = \sum_{i=1}^{r}\beta_i(\theta(k))\left[N_i z(k) + G_i u(k) + L_i y(k)\right] & \text{(9a)} \\ \hat{x}(k) = z(k) - Fy(k),\ i = 1,\ 2,\ ...,\ r, & \text{(9b)} \end{cases}$$

where $\hat{x}(k)$ is the estimated state variable of $x(k)$, $z(k)$ is the state variable, $u(k)$ is the input signal and $y(k)$ is the output signal. The matrices $N_i \in \Re^{n\times n}$, $G_i \in \Re^{n\times m}$, $L_i \in \Re^{n\times p}$, and $F \in \Re^{n\times p}$ are the observer gains to be found later. $\beta_i(\theta(k))$ is dependent on the states and is measurable. Let us define the estimation error of the state variables as

$$e(k) = \hat{x}(k) - x(k), \tag{10}$$

Substituting (8b) and (9b) into (10) leads to

$$e(k) = z(k) - FCx(k) - x(k) = z(k) - Mx(k) \tag{11}$$

where $M = I + FC.$

Lemma 1 [45]*:* Take any two matrices $A \in \Re^{m\times n}$ with $m \ge n$, $B \in \Re^{k\times n}$ and suppose that $BA^{+}A = B$. Then any matrix of the form $X = BA^{+} + Y(I - AA^{+})$ is a general solution of $XA = B$, where $Y \in \Re^{k\times m}$ is an arbitrary matrix.

Lemma 2 [46]: For any real matrices X and Y with appropriate dimensions, the following property holds for any positive scalar ε: $X^TY + Y^TX \le \varepsilon X^TX + \varepsilon^{-1}Y^TY$.

Theorem 1: Consider the system (8) with Assumption 1 and Assumption 2, the estimation error (10) with the observer (9) approaches to zero asymptotically if there exist matrices N_i, G_i, L_i, F and symmetric matrices $P_i > 0$ such that the following conditions hold:

$$N_i M - MA_i + L_i C = 0, \tag{12}$$
$$G_i - MB_i = 0, \tag{13}$$
$$MD = 0, \tag{14}$$
$$M = I + FC, \tag{15}$$
$$N_i^T P_l N_i - P_i < 0 \tag{16}$$

where $i,\ l = 1,\ 2,\ ...,\ r$.

Proof: From (11), we have

$$e(k+1) = z(k+1) - Mx(k+1). \tag{17}$$

Substituting (8a) and (9a) into (17), one obtains

$$\begin{aligned} e(k+1) &= \sum_{i=1}^{r}\beta_i(\theta(k))[N_i z(k) + G_i u(k) + L_i y(k)] \\ &- M\left\{\sum_{i=1}^{r}\beta_i(\theta(k))[A_i x(k) + B_i u(k) + D\omega_i(k)]\right\} \\ &= \sum_{i=1}^{r}\beta_i(\theta(k))[N_i e(k) + (N_i M - MA_i + L_i C)x(k) \\ &+ (G_i - MB_i)u(k) - MD\omega_i(k)] \end{aligned} \tag{18}$$

If conditions (12)-(14) hold, then (18) is equivalent to

$$e(k+1)=\sum_{i=1}^{r}\beta_i(\theta(k))[N_i e(k)] \tag{19}$$

Select a non-common quadratic Lyapunov function as

$$V(e(k))=\sum_{i=1}^{r}\beta_i(\theta(k))e^T(k)P_i e(k)\text{, then}$$
$$\Delta V(e(k))=V(e(k+1))-V(e(k))=\sum_{i=1}^{r}\beta_i(\theta(k+1))e^T(k+1)$$
$$\times P_i e(k+1)-\sum_{i=1}^{r}\beta_i(\theta(k))e^T(k)P_i e(k) \tag{20}$$

From (4), it can be easy to infer that

$$\beta_i(\theta(k+1))\geq 0 \text{ and } \sum_{i=1}^{r}\beta_i(\theta(k+1))=1 \tag{21}$$

According to (19), (20) and (21),

$$\Delta V(e(k))=\sum_{i=1}^{r}\beta_i(\theta(k+1))\left[\sum_{i=1}^{r}\beta_i(\theta(k))[N_i e(k)]\right]^T$$
$$\times P_i\left[\sum_{i=1}^{r}\beta_i(\theta(k))[N_i e(k)]\right]-\sum_{i=1}^{r}\beta_i(\theta(k))e^T(k)P_i e(k)$$
$$=\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\beta_i(\theta(k))\beta_j(\theta(k))\beta_l(\theta(k+1))e^T(k)[N_j^T P_l N_i-P_i]e(k). \tag{22}$$

$$\Delta V(e(k))=\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\beta_i(\theta(k))\beta_j(\theta(k))\beta_l(\theta(k+1))$$
$$\times e^T(k)[N_j^T P_l N_i-P_i]e(k)=\sum_{i=1}^{r}\beta_l(\theta(k+1))\Pi_l \tag{23}$$

where

$$\Pi_l=\sum_{i=1}^{r}\sum_{j=1}^{r}\beta_i(\theta(k))\beta_j(\theta(k))e^T(k)[N_j^T P_l N_i-P_i]e(k) \tag{24}$$

From (24), one obtains that

$$\Pi_l=\sum_{i=1}^{r}\beta_i^2(\theta(k))e^T(k)[N_i^T P_l N_i-P_i]e(k)$$
$$+\sum_{i=1}^{r}\sum_{j=1,i<j}^{r}\beta_i(\theta(k))\beta_j(\theta(k))e^T(k)[N_i^T P_l N_j-P_i+N_j^T P_l N_i-P_j]e(k)$$
$$=\sum_{i=1}^{r}\beta_i^2(\theta(k))e^T(k)[N_i^T P_l N_i-P_i]e(k)+\sum_{i=1}^{r}\sum_{j=1,i<j}^{r}\beta_i(\theta(k))\beta_j(\theta(k))$$
$$\times e^T(k)[N_i^T P_l^{1/2}P_l^{1/2}N_j-P_i+N_j^T P_l^{1/2}P_l^{1/2}N_i-P_j]e(k) \tag{25}$$

Applying the Lemma 2 for (25), it infers that

$$\Pi_l\leq\sum_{i=1}^{r}\beta_i^2(\theta(k))e^T(k)[N_i^T P_l N_i-P_i]e(k)+\sum_{i=1}^{r}\sum_{j=1,i<j}^{r}\beta_i(\theta(k))$$
$$\times\beta_j(\theta(k))e^T(k)[N_i^T P_l N_i-P_i+N_j^T P_l N_j-P_j]e(k)$$

It is noted that

$$\sum_{i=1}^{r}\beta_i^2(\theta(k))e^T(k)[N_i^T P_l N_i-P_i]e(k)+\sum_{i=1}^{r}\sum_{j=1,i<j}^{r}\beta_i(\theta(k))$$
$$\times\beta_j(\theta(k))e^T(k)[N_i^T P_l N_i-P_i+N_j^T P_l N_j-P_j]e(k)$$
$$=\sum_{i=1}^{r}\beta_i(\theta(k))e^T(k)[N_i^T P_l N_i-P_i]e(k).$$

Hence, we obtain

$$\Delta V(e(k))=\sum_{l=1}^{r}\beta_l(\theta(k+1))\Pi_l\leq\sum_{l=1}^{r}\sum_{i=1}^{r}\beta_l(\theta(k+1))$$
$$\times\beta_i(\theta(k))e^T(k)[N_i^T P_l N_i-P_i]e(k) \tag{26}$$

If the condition (16) holds, it leads to $\Delta V(e(k))<0$, it means that the estimation error (10) will approach to zero asymptotically. Hence, the proof is completed.

To synthesize the observer (9), the conditions (12)-(16) must be satisfied to obtain the observer gains N_i, G_i, L_i and F. It is noted that the condition (16) is a BMI which is difficult to solve, therefore (16) needs to be transformed into an LMI so that it can be resolved easily using LMI tools. To transform BMI to LMI, the following lemma is needed.

Lemma 3 [50]: Let $S\in\Re^n$, $\Delta\in\Re^n$; Ω and Φ are the matrices with compatible dimensions, the following two statements are equivalent:

i) There exists a positive symmetric matrix P such that $\Phi^T P\Phi-\Delta<0$

ii) There exist positive symmetric matrices P and Ω such that

$$\begin{bmatrix}-\Delta & * \\ \Omega\Phi & -\Omega-\Omega^T+P\end{bmatrix}<0.$$

By adding the slack variable X and applying Lemma 3 for (16), (16) is equivalent to

$$\begin{bmatrix}-P_i & * \\ XN_i & -X-X^T+P_l\end{bmatrix}<0 \tag{27}$$

From (14) and (15), it implies that

$$MD=(I+FC)D=0$$
$$D+FCD=0 \tag{28}$$

Because of Lemma 1, the general solution of (28) is expressed in the following

$$F=-D(CD)^{+}+Y(I-(CD)(CD)^{+}) \tag{29}$$

where Y is an arbitrary matrix with compatible dimensions. It is noted that the matrices C and D must satisfy Assumption 2 to guarantee the existence of (29). Next, define

$$U=-D(CD)^{+}, \tag{30}$$
$$V=I-(CD)(CD)^{+}. \tag{31}$$

Substituting (30) and (31) into (29) yields

$$F = U + YV \,. \tag{32}$$

Let

$$K_i = L_i + N_i F \,. \tag{33}$$

From (12) and (33), it follows that

$$N_i = MA_i - K_i C \tag{34}$$

$$L_i = K_i (I + CF) - MA_i F \,. \tag{35}$$

Let (15), (32) and (34) be substituted into (27), then (27) is equivalent to

$$\begin{bmatrix} -P_i & * \\ X((I + (U + YV)C)A_i - K_i C) & -X - X^T + P_l \end{bmatrix} < 0 \tag{36}$$

Define

$$Q = XY \tag{37}$$

and

$$\bar{K}_i = XK_i \tag{38}$$

Substituting (37) and (38) into (36), the inequality (36) becomes

$$\begin{bmatrix} -P_i & * \\ X((I + UC)A_i) + Q(VCA_i) - \bar{K}_i C & -X - X^T + P_l \end{bmatrix} < 0 \,. \tag{39}$$

It is seen that (39) is an LMI which can be solved easily using the Matlab LMI toolbox. Considering the other conditions (13), (32), (34) and (35) together, the observer gains F, G_i, N_i, L_i are obtained, and the observer (9) is synthesized. The following is a brief procedure of the observer synthesis.

Step 1: Check whether the matrices C and D fulfill the Assumption 2 or not. If not, this synthesis does not work. If yes, go to the next step.
Step 2: Obtain U and V from conditions (30) and (31), respectively.
Step 3: Solve LMI (39) to find $P_i, X, Q,$ and $\bar{K}_i$.
Step 4: Y and K_i are obtained from (37) and (38). M and F are obtained from (15) and (32), respectively. From (13), (34) and (35), the matrices G_i, N_i, and L_i are obtained. The observer (9) is synthesized.

B. Observer Synthesis with Estimated Premise Variables

In this subsection, we consider the case that premise variables of (9) depend on the un-measurable states. It is obvious that it is impossible to construct the observer (9) when the premise variables are unavailable. In this case, the premise variables in (9a) should be estimated. Thus, the result in subsection III-A must be modified as below.

Consider an observer whose premise variables are dependent on estimated state variables for the system (8) as follows

$$\begin{cases} z(k+1) = \sum_{i=1}^{r} \beta_i(\hat{\theta}(k)) \left[N_i z(k) + G_i u(k) + L_i y(k) \right] & \text{(40a)} \\ \hat{x}(k) = z(k) - Fy(k), \; i = 1, \; 2, \; ..., \; r. & \text{(40b)} \end{cases}$$

where $\hat{\theta}(k)$ is the estimation of $\theta(k)$.
The estimation error is defined as

$$e(k) = \hat{x}(k) - x(k) \tag{41}$$

According to (8b), (40b), and (41), this yields

$$e(k) = z(k) - Mx(k) \tag{42}$$

where $M = I + FC$.

From (42), it infers that

$$e(k+1) = z(k+1) - Mx(k+1) \,. \tag{43}$$

Substituting (8a) and (40a) into (43) obtains the following result

$$\begin{aligned} & e(k+1) = \sum_{i=1}^{r} \beta_i(\hat{\theta}(k))[N_i z(k) + G_i u(k) + L_i y(k)] \\ & -M \left\{ \sum_{i=1}^{r} \beta_i(\theta(k))[A_i x(k) + B_i u(k) + D\omega_i(k)] \right\} \\ & = \sum_{i=1}^{r} \beta_i(\hat{\theta}(k))[N_i(e(k)) + (N_i M - MA_i + L_i C)x(k) + (G_i - MB_i)u(k)] \\ & + M \sum_{i=1}^{r} \left[\beta_i(\hat{\theta}(k)) - \beta_i(\theta(k)) \right] [A_i x(k) + B_i u(k)] - \sum_{i=1}^{r} \beta_i(\theta(k))[MD\omega_i(k)] \end{aligned} \tag{44}$$

Define $\Lambda(\hat{\theta}, \theta, x, u) = \sum_{i=1}^{r} \left[\beta_i(\hat{\theta}(k)) - \beta_i(\theta(k)) \right] [A_i x(k) + B_i u(k)]$.

For simplicity, let us employ $\Lambda(.)$ instead of $\Lambda(\hat{\theta}, \theta, x, u)$ in the consequent. Hence, (44) is rewritten as follows

$$\begin{aligned} & e(k+1) = \sum_{i=1}^{r} \beta_i(\hat{\theta}(k)) \left[N_i(e(k)) + (N_i M - MA_i + L_i C)x(k) \right. \\ & \left. + (G_i - MB_i)u(k) \right] + M\Lambda(.) - \sum_{i=1}^{r} \beta_i(\theta(k))[MD\omega_i(k)] \end{aligned} \tag{45}$$

Assumption 3 [26]: In the above defined function $\Lambda(.)$, which has a positive constant α satisfying $\|\Lambda(.)\| \le \alpha \|e(k)\|$, *i.e.*, $\Lambda(.)$ is the Lipschitz in $e(k)$.
Remark 3: This assumption is needed in the derivation of Theorem 2 and also seen in many papers such as [26], [35], [39]-[43] and [51].

Theorem 2: Consider the system (8) with premise variables dependent on the un-measurable states. For a given $\alpha > 0$, the estimation error $e(k)$ (41) with the observer (40) converges to zero asymptotically, if there exist matrices N_i, G_i, L_i, F, scalar $\lambda > 0$ and symmetric matrices $P_i > 0$, such that the following conditions hold,

$$N_i M - MA_i + L_i C = 0, \quad (46)$$

$$G_i - MB_i = 0, \quad (47)$$

$$MD = 0, \quad (48)$$

$$M = I + FC, \quad (49)$$

$$\begin{bmatrix} H_{il}^{(11)} & 0 \\ 0 & \Sigma_l^{(22)} \end{bmatrix} < 0 \quad (50)$$

Where

$$H_{il}^{(11)} = 2N_i^T P_l N_i - P_i + \lambda\alpha^2 I, \quad (51)$$

$$\Sigma_l^{(22)} = 2M^T P_l M - \lambda I, \quad (52)$$

$i,\ l = 1,\ 2,\ \ldots,\ r$.

Proof: If the conditions (46)-(48) of Theorem 2 hold, (45) is written as follows

$$e(k+1) = \sum_{i=1}^{r} \beta_i(\hat{\theta}(k))[N_i e(k)] + M\Lambda(.) \quad (53)$$

Select the non-common quadratic Lyapunov function

$V(e(k)) = \sum_{i=1}^{r} \beta_i(\hat{\theta}(k)) e^T(k) P_i e(k)$, then it follows that

$$\Delta V(e(k)) = V(e(k+1)) - V(e(k))$$
$$= \sum_{i=1}^{r} \beta_i(\hat{\theta}(k+1)) e^T(k+1) P_i e(k+1) - \sum_{i=1}^{r} \beta_i(\hat{\theta}(k)) e^T(k) P_i e(k) \quad (54)$$

Substituting (53) into (54) yields

$$\Delta V(e(k)) = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r} \beta_i(\hat{\theta}(k))\beta_j(\hat{\theta}(k))\beta_l(\hat{\theta}(k+1)) e^T(k)$$
$$[N_j^T P_l N_i - P_i] e(k) + \sum_{i=1}^{r}\sum_{l=1}^{r} \beta_i(\hat{\theta}(k))\beta_l(\hat{\theta}(k+1)) e^T(k)$$
$$\times [N_i^T P_l M]\Lambda(.) + \sum_{i=1}^{r}\sum_{l=1}^{r} \beta_i(\hat{\theta}(k))\beta_l(\hat{\theta}(k+1))\Lambda^T(.)$$
$$\times [M^T P_l N_i] e(k) + \sum_{i=1}^{r} \beta_l(\hat{\theta}(k+1))\Lambda^T(.)[M^T P_l M]\Lambda(.). \quad (55)$$

Based on Lemma 2, (55) is equivalent to

$$\Delta V(e(k)) \le \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r} \beta_i(\hat{\theta}(k))\beta_j(\hat{\theta}(k))\beta_l(\hat{\theta}(k+1))$$
$$\times e^T(k)[2N_j^T P_l N_i - P_i] e(k) + \sum_{i=1}^{r} \beta_l(\hat{\theta}(k+1))\Lambda^T(.)[2M^T P_l M]$$
$$\times \Lambda(.) - \lambda\Lambda^T(.)\Lambda(.) + \lambda\Lambda^T(.)\Lambda(.) \quad (56)$$

According to Assumption 3, (56) is written as

$$\Delta V(e(k)) \le \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r} \beta_i(\hat{\theta}(k))\beta_j(\hat{\theta}(k))\beta_l(\hat{\theta}(k+1)) e^T(k)[2N_j^T P_l N_i$$
$$-P_i + \lambda\alpha^2 I] e(k) + \sum_{l=1}^{r} \beta_l(\hat{\theta}(k+1))\Lambda^T(.)\left[2M^T P_l M - \lambda I\right]\Lambda(.)$$

Follow the steps (23)-(26) of III-A, we have

$$\Delta V(e(k)) \le \sum_{i=1}^{r}\sum_{l=1}^{r} \beta_i(\hat{\theta}(k))\beta_l(\hat{\theta}(k+1)) e^T(k)[2N_i^T P_l N_i - P_i$$
$$+\lambda\alpha^2 I] e(k) + \sum_{l=1}^{r} \beta_l(\hat{\theta}(k+1))\Lambda^T(.)\left[2M^T P_l M - \lambda I\right]\Lambda(.)$$

$$\Delta V(e(k)) \le \sum_{i=1}^{r}\sum_{l=1}^{r} \beta_i(\hat{\theta}(k))\beta_l(\hat{\theta}(k+1))\left[e^T(k) \quad \Lambda^T(.)\right]$$
$$\times \begin{bmatrix} H_{il}^{(11)} & 0 \\ 0 & \Sigma_l^{(22)} \end{bmatrix} \begin{bmatrix} e(k) \\ \Lambda(.) \end{bmatrix} \quad (57)$$

where $H_{il}^{(11)} = 2N_i^T P_l N_i - P_i + \lambda\alpha^2 I$ and $\Sigma_l^{(22)} = 2M^T P_l M - \lambda I$.

From (57), it is obvious that if the condition (50) holds, (57) implies that $\Delta V(e(k)) < 0$ and the estimation error (41) is guaranteed to approach to zero asymptotically. Hence, the proof is completed.

The condition (50) of Theorem 2, however, is a BMI which should be transformed into LMIs so that it can be solved easily using LMI tools. By adding a slack variable X with appropriate dimensions and employing the Lemma 3 for the first-diagonal-block $H_{il}^{(11)}$ and the second- diagonal-block $\Sigma_l^{(22)}$ of (50), we have

$$H_{il}^{(11)} = \begin{bmatrix} -P_i + \lambda\alpha^2 I & * \\ XN_i & -X - X^T + 2P_l \end{bmatrix} \quad (58)$$

$$\Sigma_l^{(22)} = \begin{bmatrix} -\lambda I & * \\ XM & -X - X^T + 2P_l \end{bmatrix}. \quad (59)$$

It is similar to (28)-(39), the BMI (50) is transformed into LMI (60) as follows,

$$\begin{bmatrix} H_{il}^{(11)} & 0 \\ 0 & \Sigma_l^{(22)} \end{bmatrix} < 0 \quad (60)$$

where

$$H_{il}^{(11)} = \begin{bmatrix} -P_i + \lambda\alpha^2 I & * \\ X((I+UC)A_i) + Q(VCA_i) - \bar{K}_i C & -X - X^T + 2P_l \end{bmatrix},$$

$$\Sigma_l^{(22)} = \begin{bmatrix} -\lambda I & * \\ X(I+UC) + Q(VC) & -X - X^T + 2P_l \end{bmatrix}.$$

The matrices U, V, F, M, K_i, $\bar{K}_i$, Q, N_i, and L_i are similarly defined in the subsection III-A.

Let us summarize the above derivation to the following procedure for the observer synthesis.

Step 1 and *Step 2:* The same as the first two steps in subsection III-A, respectively.

Step 3: Solve LMI (60) with given $\alpha > 0$ to find P_i, X, Q, $\bar{K}_i$ and λ.

Step 4: The same as the Step 4 in subsection III-A. Then the observer (40) is synthesized.

Remark 4: Based on Assumption 3, α is given, such that the conditions (46)-(50) are solved to obtain the observer gains of the observer (40). This assumption has been used in many previous papers such as [26], [35], [39]-[43] and [51]. Nevertheless, we do not actually know the given α satisfies Assumption 3 or not before we do our design process. Thus, after finishing the synthesis of the observer (40), we should verify the correctness of Assumption 3. If Assumption 3 is verified, the observer synthesis is successful. Otherwise, another α should be chosen until a feasible solution is found. However, how to find a suitable α is still an open problem worthy of future study.

Remark 5: It is noted that the matrix sizes and variable numbers of both LMIs (39) and LMIs (60) are smaller and fewer than those in the theorems in [39] and [40]. Therefore, the LMIs (39) and (60) are much easier to solve by Matlab tool than those in [39] and [40].

IV. Illustrative Example

In this example, a practical application of a system for DC controlling an inverted pendulum via a gear train [48], [49] is considered. Figure 1 shows the structure of the system and the armature-controlled DC motor is depicted in Fig. 2.

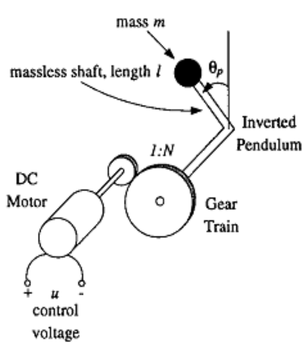

Fig. 1. The structure.

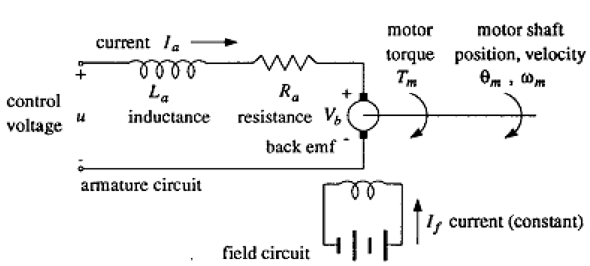

Fig. 2. Schematic of an armature-controlled DC motor.

The continuous time nonlinear model for the above system is described as follows [48].

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{g}{l}\sin(x_1) + \frac{NK_m}{ml^2}x_3 \\ -\frac{K_b N}{L_a}x_2 - \frac{R_a}{L_a}x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_a} \end{bmatrix} u \tag{61}$$

where $x(t) = [x_1,\ x_2, x_3]^T = [\theta_p, \dot{\theta}_p, I_a]^T$. K_m is the motor torque constant, K_b is the back EMF constant and N is the gear ratio. The parameters of the system are $g = 9.8\ m/s^2$, $l = 1\,m$, $m = 1\,kg$, $N = 10$, $K_m = 0.1\ N\text{m/A}$, $K_b = 0.1\ Vs/rad$, $R_a = 1\,\Omega$ and $L_a = 100\ m\text{H}$.

Transforming continuous time system into discrete time system with sampled time *T=0.1* yields

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} x_1(k) + Tx_2(k) \\ T\frac{g}{l}\sin(x_1(k)) + x_2(k) + T\frac{NK_m}{ml^2}x_3(k) \\ -T\frac{K_b N}{L_a}x_2(k) - T\frac{R_a}{L_a}x_3(k) + x_3(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ T\frac{1}{L_a} \end{bmatrix} u(k) \tag{62}$$

Employing the nonlinear-sector method to transform the system (62) into a discrete time T-S fuzzy system and assuming that this system is affected by uncertainties, we obtained the system as (3) with parameters and uncertainties as follows.

$$A_1 = \begin{bmatrix} 1 & 0.1 & 0 \\ 0.98 & 1 & 0.1 \\ 0 & -1 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0.1 & 0 \\ 0 & 1 & 0.1 \\ 0 & -1 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \Delta A_1 = \begin{bmatrix} -0.044\xi_2(k) & 0.08\xi_1(k) & 0.44\xi_2(k) \\ 0.11\xi_2(k) & 0.2\xi_1(k) & 1.1\xi_2(k) \\ -0.099\xi_2(k) & -0.16\xi_1(k) & -0.88\xi_2(k) \end{bmatrix},$$
$$\Delta A_2 = \begin{bmatrix} -0.44\xi_1(k) & 0.16\xi_2(k) & 0.12\xi_1(k) \\ -1.1\xi_1(k) & 0.4\xi_2(k) & 0.3\xi_1(k) \\ -0.88\xi_1(k) & -0.32\xi_2(k) & -0.24\xi_1(k) \end{bmatrix}$$
$$\Delta B_1 = \begin{bmatrix} 0.48\xi_1(k) \\ 1.2\xi_1(k) \\ 0.96\xi_1(k) \end{bmatrix}, \Delta B_2 = \begin{bmatrix} 0.44\xi_2(k) \\ 1.1\xi_2(k) \\ -0.88\xi_2(k) \end{bmatrix}. \tag{63}$$

$\xi_1(k)$ and $\xi_2(k)$ are random noises obtained from a normal distribution over [-1,1].

The normalized weight functions are expressed as follows

$$\beta_1(x_1(k)) = \begin{cases} \dfrac{\sin(x_1(k))}{x_1(k)}, & x_1(k) \neq 0 \\ 1, & x_1(k) = 0 \end{cases}; \qquad \beta_2(x_1(k)) = 1 - \beta_1(x_1(k)).$$

On the basis of the Assumption 1, (3) is written in the form of (8) with $D = \begin{bmatrix} 0.4 & 1 & -0.8 \end{bmatrix}^T$.

Firstly, the case of measurable premise variables is considered. We choose $C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$, it means that x_1 is measurable; x_2 and x_2 are unavailable. Because premise variables depend on x_1, hence, the premise variables are available. Aditionally, $Rank(CD) = Rank(D) = 1$ satisfies Assumption 2. From Theorem 1 and observer synthesis procedures in Section III-A, the observer gains of observer (9) are found as

$$F = \begin{bmatrix} -1 \\ -2.5 \\ 2 \end{bmatrix}, \; G_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, L_1 = \begin{bmatrix} 0 \\ 0.155 \\ 0 \end{bmatrix}, L_2 = \begin{bmatrix} 0 \\ -0.825 \\ 0 \end{bmatrix},$$

$$N_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.75 & 0.1 \\ 0 & -0.8 & 0.0 \end{bmatrix}, \; N_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.75 & 0.1 \\ 0 & -0.8 & 0 \end{bmatrix} G_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

The simulation results are shown as follows.

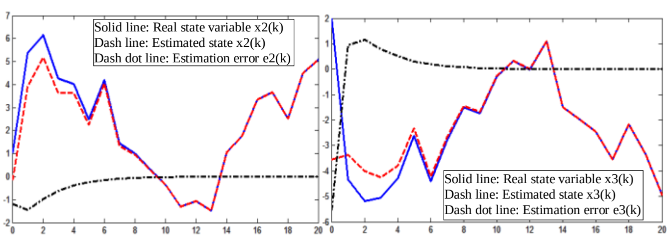

Fig. 3. Real state, estimated state and estimation error with measurable premise variables

It is noted that the state x_1 is available, therefore we do not need to show its estimation result in Fig. 3.

Next, consider the case with estimated premise variables for the same system (62) and uncertainties (63). Let the output matrix be $C = \begin{bmatrix} 0 & -1 & 1 \end{bmatrix}$, x_1 is unavailable. Because the premise variables are dependent on x_1, that leads the premise variables to be unknown. Therefore, the premise variables need to estimate. On the basis of the process of observer synthesis and Theorem 2 in Section III-B, with a given $\alpha = 0.3$, the observer gains are derived as follows.

$$N_1 = \begin{bmatrix} 0.7822 & -1.2697 & 0.9031 \\ 0.4356 & 0.0810 & -0.1477 \\ 0.4356 & 0.0790 & -0.1456 \end{bmatrix}, L_2 = \begin{bmatrix} -0.0407 \\ 0.0815 \\ 0.0815 \end{bmatrix}$$

$$N_2 = \begin{bmatrix} 1 & 0.4339 & -0.8005 \\ 0 & 0.0871 & -0.1538 \\ 0 & 0.0988 & -0.1655 \end{bmatrix}, \; L_1 = \begin{bmatrix} 0.0077 \\ -0.0153 \\ -0.0153 \end{bmatrix}.$$

$$F = \begin{bmatrix} 0.2222 \\ 0.5556 \\ -0.4444 \end{bmatrix} G_1 = \begin{bmatrix} 0.2222 \\ 0.5556 \\ 0.5556 \end{bmatrix}, G_2 = \begin{bmatrix} 0.2222 \\ 0.5556 \\ 0.5556 \end{bmatrix},$$

Then, the simulation results are shown in the following figures.

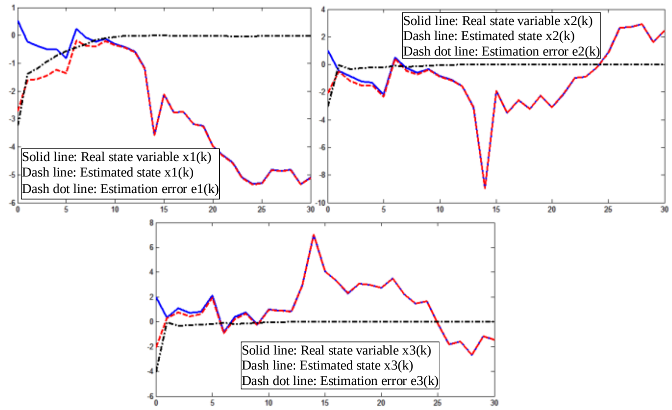

Fig. 4. Real state, estimated states and estimation errors with un-measurable premise variables.

After completing observer synthesis, the satisfaction of the Assumption 3 must be verified.

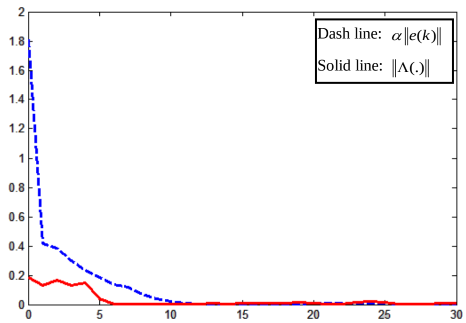

Fig. 5 Satisfaction of the Assumption 3

The simulation results including real states $x(k)$, estimated states $\hat{x}(k)$ and estimation errors $e(k)$ for both measurable and unmeasurable premise variables cases are depicted in Fig. 3 and Fig. 4, respectively. Obviously, two figures show that the estimated state variables approach to the real state variables asymptotically. Thus, the angle θ_p, velocity angle $\dot{\theta}_p$ and current I_a of (61) are estimated successfully in the both cases. Furthermore, Fig 5 shows that the values of $\|\Lambda(.)\|$ are always smaller than values of $\alpha\|e(k)\|$, it implies that the Assumption 3 is fulfilled and $\alpha = 0.3$ is acceptable.

Remark 6: Since there is no information about the uncertainty bounds, the methods of the papers [28]-[37] cannot be used to solvethe problem.

V. Conclusion

A new method to synthesize an observer for discrete time T-S fuzzy system with uncertainties has been developed in this paper. The proposed method can eliminate the influences of uncertainties and estimate the state variables asymptotically when the upper bounds of uncertainties are unavailable. In addition, the non-common quadratic Lyapunov functions are employed to reduce the conservatism of the main results. The conditions in term of LMIs for observer synthesis in both measurable and estimated premise variables are given. A practical example is presented to show the success of the proposed method. However, in this paper, the rechecking of Assumption 3 after observer synthesis is needed. The holding of Assumption 3 before the design work may be an open problem for future work.

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